PG – 365

II Semester M.Sc. Degree Examination, June 2015
(CBCS)
MATHEMATICS
M 204 T : Partial Differential Equations

Time : 3 Hours   Max. Marks : 70

Instructions : *1) **All** questions have **equal** marks.*
*2) Answer **any five** questions.*

1. a) Give the geometrical interpretation of a quasilinear first order partial differential equation. Also derive the characteristic equations for the same.

   b) Find the integral surface of the partial differential equation

   $xp + yq = z$

   which contains the circle $x^2 + y^2 + z^2 = 4$ and $x + y + z = 2$.

   c) Use method of characteristics to solve $(y - u)\, u_x + (u - x)\, u_y = x - y$ with condition $u = 0$ on $xy = 1$. **(5+4+5)**

2. a) Find the solution of the equation $u(x + y)\, u_x + u(x - y)\, u_y = (x^2 + y^2)$ with the Cauchy data $u = 0$ on $y = 2x$.

   b) Solve : $p^2 + q + u = 0$ with $u(x, 0) = x$. **(7+7)**

3. a) Transform the standard second order hyperbolic partial differential equation to its canonical form.

   b) Classify the equation :

   $\sin^2 x\, u_{xx} + \sin 2x\, u_{xy} + \cos^2 x\, u_{yy} = x$ and hence reduce it to its canonical form. **(8+6)**

4. a) Solve the following by the Monge's method :

   $(1 + q)^2\, r - 2(1 + p + q + pq)\, s + (1 + p)^2\, t = 0.$

   b) Verify whether the following equation

   $x^3 u_{xx} + (y^2 + yz)\, u_{yy} + 3x^2 u_x + (2y + z)\, u_y = 0$ is self-adjoint or not. **(8+6)**

5. a) Solve by variable separable method the following IBVP :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \ 0 < x < 1 \ ; \ t > 0$$

Subject to

$$\left.\begin{array}{l} u(x, 0) = f(x) \\ \frac{\partial u}{\partial t}(x, 0) = g(x) \end{array}\right\} \ 0 \le x \le 1$$

$$u(0, t) = 0 = u(1, t), \ t \in R$$

b) Show that a variable separable solution of wave equation in spherical coordinates leads to a Legendre differential equation. **(7+7)**

6. State and prove the Dirichlet problem in a circular region. **14**

7. a) Solve by appropriate Fourier transform the following IBVP :

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}, \ 0 \le x < \infty \ ; \ t \ge 0$$

Subject to

$$u(x, 0) = f(x), \ 0 \le x < \infty$$

$$\frac{\partial u}{\partial x}(0, t) = 0, \ \ t > 0$$

b) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \ 0 < x < 1 \ ; \ t \ge 0$

Subject to $u(x, 0) = 0 \ ; \ 0 < x \le 1$

$$\left.\begin{array}{l} u(0, t) = 0 \\ u(1, t) = f(t) \end{array}\right\}, \ t > 0.$$ **(7+7)**

8. Find the Green's function for the following :

a) $\frac{\partial u}{\partial t} - k\frac{\partial^2 u}{\partial x^2} = f(x)\ \delta(t)\ ;\ -\infty < x < \infty,\ t > 0$

Subject to

$u(x, 0) = 0\ ;\ -\infty < x < \infty$

b) $\frac{\partial^2 u}{\partial t^2} - C_1^2\frac{\partial^2 u}{\partial x^2} = Q_1(x), -\infty < x < \infty,\ t \geq 0$

Subject to

$$\left.\begin{aligned} u(x,0) &= 0 \\ \frac{\partial u}{\partial t}(x,0) &= 0 \end{aligned}\right\};\ -\infty < x < \infty,$$

$$\left.\begin{aligned} u &\to 0 \\ \frac{\partial u}{\partial x} &\to 0 \end{aligned}\right\} \text{ as } |x| \to \infty.$$

(7+7)